EXERCISE – IV HINTS & SOLUTIONS

Sol.1 **B**

$x^2y = 1 - y$
$xy = 1 - y$
$x^2y = xy$
$x^2y - xy = 0$
$xy(x - 1) = 0$
$xy = 0$ & $x = 1$ $\Rightarrow y = 1/2$
$x = 0 \Rightarrow y = 1$ POI P(1, 1/2)
$y = 0$ $0 = 1$ not possible Q(0, 1)
$x^2y = 1 - y$
$2xy + x^2y' = -y'$

$$y' = \left.\frac{-2xy}{1+x^2}\right|_P = \frac{-2(1)(1/2)}{1+1} = -\frac{1}{2}$$

$y'|_Q = 0$

at P tangent $y - \frac{1}{2} = -\frac{1}{2}(x - 1)$ ....(1)

at Q tangent $y - 1 = 0 \Rightarrow y = 1$ ....(2)

Intarsection of (1) & (2)

$$\frac{1}{2} = -\frac{1}{2}(x - 1)$$

$x - 1 = -1$
$x = 0$
POI (0, 1)

Sol.2 $\frac{dy}{dx} = \frac{2t+1}{2-2t}$

equation of tangent

$$y - (t + t^2) = \frac{2t+1}{2-2t}(x - (2t - t^2))$$

at (1, 1) $\Rightarrow 2t^2 + 2t - 2 = 2t^2 - t - 1$
so ; $t = 1/3$; $m = 5/4$
$t = 1$; $m \to \infty$

Sol.3 $y'(0) = \lim\limits_{h\to 0} \frac{1/h\sin h^2 - 0}{h} = 1$

$x'(0) = \lim\limits_{h\to 0} \frac{2h + h^2\sin 1/h - 0}{h} = 2$

so $m = 1/2$
T : $y = 1/2x$
N : $y = -2x$

Sol.4 **A,B**

$y = \cos(x + y)$
$y' = -\sin(x + y)$ $(1 + y')$

$$y' = -\frac{\sin(x+y)}{1+\sin(x+y)} = -\frac{1}{2}$$

$\sin(x + y) = 1$
$\cos(x + y) = 0$
$y_1 = \cos(x_1 + y_1) = 0$
$\sin(x_1 + y_1) = 1 \Rightarrow \sin x_1 = 1$

$$x_1 = \frac{\pi}{2}, -\frac{3\pi}{2}$$

Point $\left(\frac{\pi}{2}, 0\right)$ and $\left(-\frac{3\pi}{2}, 0\right)$

This two points satisfies

Sol.5 $\frac{dy}{dx} = \frac{a(\cos t - \cos t + t\sin t)}{a(-\sin t + \sin t + t\cos t)} = -\tan t$

equation normal

$$y - a(\sin t - t\cos t) = -\frac{\cos x}{\sin x}(x - a\cos t - a\,t\sin t)$$

$\Rightarrow y\sin t - a\sin^2 t + at\sin t\cos t$
$= -x\cos t + a\cos^2 t + a + \sin t\cos t$
$\Rightarrow x\cos t + y\sin t = a$
tangent to given circle.

Sol.6 $x_1^3 + y_1^3 = a^3$ & $x_2^3 + y_2^3 = a^3$

subtract $\Rightarrow (x_2^3 - x_1^3) + (y_2^3 - y_1^3) = 0$ .....(i)

$$\left.\frac{dy}{dx}\right|_{(x_1, y_1)} = -\frac{x_1^2}{x_1^2} \quad .....(ii)$$

$$\text{also slope} = \frac{y_2 - y_1}{x_2 - x_1} = -\left(\frac{x_2^2 + x_1^2 + x_1x_2}{y_2^2 + y_1^2 + y_1y_2}\right) \quad ..(iii)$$

$$\text{from (ii) \& (iii)} -\frac{x_1^2}{y_1^2} = -\left(\frac{x_2^2 + x_1^2 + x_1x_2}{y_2^2 + y_1^2 + y_1y_2}\right)$$

on solving : $\frac{x_2}{x_1} + \frac{y_2}{y_1} = -1$

Sol.7 $y = x^2 - x^3$

Let $P(t_1, t_1^2 - t_1^3)$

& $Q(t_2, t_2^2 - t_2^3)$

$$\frac{dy}{dx} = 2x - 3x^2|_p = 2t_1 - 3t_1^2$$

$2t_1 - 3t_1^2 = M_{PQ}$

$$2t_1 - 3t_1^2 = \frac{t_2^2 - t_2^3 - t_1^2 + t_1^3}{t_2 - t_1}$$

$2t_1 - 3t_1^2 = (t_2 + t_1) - (t_2^2 + t_1^2 + t_1 t_2)$

$\Rightarrow \quad t_2 + 2t_1 - 1 = 0 \quad$ ......(1)

Now $\quad 2h = t_1 + t_2 \quad$ ......(2)

$2k = t_1^2 - t_1^3 + t_2^2 - t_2^3 \quad$ ......(3)

from (1) & (2)

$t_1 = 1 - 2h$

& $\quad t_2 = 3 - 4h$

put the values is equation (3)

$k = 1 - 9h + 28h^2 - 28h^3$

Locus $\Rightarrow$ $y = 1 - 9x + 28x^2 - 28x^3$

Sol.8 Let the point $P(c \cos^3\theta, c \sin^3\theta)$ lie on the curve $x^{2/3} + y^{2/3} = c^{2/3}$

$$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}} = -\tan\theta$$

tangent at point P

$y - c\sin^3\theta = -\tan\theta\,(x - c\cos^3\theta)$

$y = -\tan\theta\, x + c\tan\theta\cos^3\theta + c\sin^3\theta$

Now apply cond^n of tangency of ellipse

$c^2 = b^2 + a^2 m^2$

$(c\tan\theta\cos^3\theta + c\sin^3\theta)^2 = b^2 + a^2\tan^2\theta$

$\Rightarrow$ $c = a + b$

Sol.9 for P & Q : $t_1 t_2 = -1$

$$P = (at^2, at^3),\ Q = \left(\frac{a}{t^2}, -\frac{a}{t^3}\right)$$

$$\left.\begin{aligned} &T_1 : y - at^3 = \frac{3t}{2}(x - at^2) \\ &T_2 : y + \frac{a}{t^3} = -\frac{3t}{2}\left(x - \frac{a}{t^2}\right)\end{aligned}\right\}\ \text{solve these equation for point of intersection}$$

Sol.10 Let point A & B are (t_1, t_1^2), (t_2, t_2^2)

$T_1 : y = 2t_1x - t_1^2 \quad$ ....(i)

$T_2 : y = 2t_2x - t_2^2 \quad$ ....(ii)

$$\text{On solving : } C = \left(\frac{t_1 + t_2}{2}, t_1 t_2\right)$$

$$\text{Mid point of A \& B} = \left(\frac{t_1 + t_2}{2}, \frac{t_1^2 + t_2^2}{2}\right)$$

Find the length of median & also

$\perp^r$ distance from A to median.

Now area = 2.(area of half Δ).

Sol.11 **(a)** Take log & $\left.\dfrac{dy}{dx}\right|_{(x_1, y_1)} = \dfrac{-y_1}{nx_1}$

$$L_{SN} = y_1\left(\frac{-y_1}{nx_1}\right) = \frac{-y_1^2 . y^n}{n . a^{n+1}} = -\frac{y_1^{n+2}}{a^{n+1} . n}$$

$n + 2 = 0 \ \Rightarrow\ n = -2$

(b) $m = \left.\dfrac{dy}{dx}\right|_{(x_1, y_1)} = \dfrac{2ax_1}{x_1^2 - a^2}$

$$L_T + L_{ST} = y_1\sqrt{1 + \frac{1}{m^2}} + \frac{y_1}{m}$$

$$= y_1\left[\frac{\sqrt{x_1^4 + a^4 + 2a^2x_1^2}}{2ax_1} + \frac{x_1^2 - a^2}{2ax_1}\right]$$

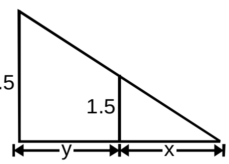
$$= \frac{y_1}{2ax_1}(x_1^2 + a^2 + x_1^2 - a^2) = \frac{x_1 y_1}{2a}.$$

Sol.12 $\left(\dfrac{dy}{dx}\right)_{C_1} . \left(\dfrac{dy}{dx}\right)_{C_2} = -1$

$$\left(\frac{1}{2y_1}\right) . \left(-\frac{y_1}{x_1}\right) = -1 \Rightarrow x_1 = 1/2$$

from Ist curve : $y_1 = \pm\dfrac{1}{\sqrt{2}}$ $\quad K = \pm\dfrac{1}{2\sqrt{2}}$

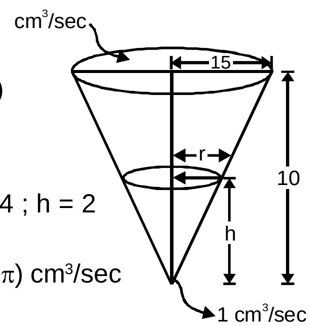
Sol.13 $\dfrac{dy}{dt} = -4$ km/hr



also $\dfrac{x+y}{4\cdot 5} = \dfrac{x}{1\cdot 5}$

$\Rightarrow$ $y = -2x$ so $\dfrac{dx}{dt} = 2$ km/hr.

Sol.14 $\dfrac{r}{h} = \dfrac{15}{10} \Rightarrow r = \dfrac{3h}{2}$ also $v = \dfrac{1}{3} pr^2$ R $= \dfrac{3\pi}{4} . h^3$



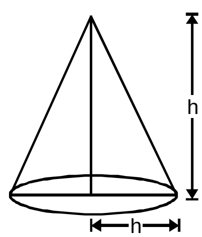
$\dfrac{dv}{dt} = \dfrac{9\pi h^2}{4} . \dfrac{dh}{dt}$ .....(i)

$\dfrac{dv}{dt} = c - 1$ & $\dfrac{dh}{dt} = 4$; $h = 2$

put in (i) $c = (1 + 36\pi)$ cm^3/sec

Sol.15 $h = 1/6r \Rightarrow r = 6h$

& volume $v = \dfrac{1}{3} \pi r^2 h$



$\dfrac{dv}{dt} = \dfrac{1}{3} \pi \dfrac{h^3}{3} \times 36 \dfrac{dh}{dt}$

$\Rightarrow$ $\dfrac{dh}{dt} = \dfrac{1}{48\pi}$ cm^3/sec.

Sol.16 $\dfrac{dr}{dt} = 10$ cm/sec, $\dfrac{dh}{dt} = -0.4$ cm/sec

$v = \pi r^2 h$, $\dfrac{dv}{dt} = 2\pi rh . \dfrac{dr}{dt} + \pi r^2 \dfrac{dh}{dt} = 0$

$\Rightarrow$ $h = 2$ cm $\qquad (r = 100)$

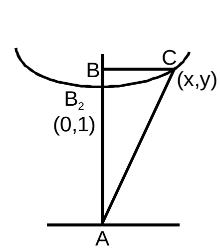
$r_1^2 h_1 = r_2^2 h_2 \Rightarrow (100)^2 . 2 = (200)^2 . h_2$

$h_2 = 1/2$ cm

$\dfrac{dv}{dt} = \pi r^2 . \dfrac{dh}{dt} + 2\pi rh . \dfrac{dr}{dt} = 0$

$\Rightarrow 200 . \dfrac{dh}{dt} = -2. \dfrac{1}{2} \times 10 \Rightarrow \dfrac{dh}{dh} = -\dfrac{1}{20}$ cm/sec

Sol.17 $y = 1 + \dfrac{7x^2}{36}$



$\dfrac{dy}{dt} = 2 = \dfrac{14x}{36} \dfrac{dx}{dt}$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{6}{7}$ (at x = 6)

{at t = 7/2 ; y = 7 ; AB = 8 so y = 8 & x = 6}

Δ = Area $= \dfrac{xy}{2} = \dfrac{x}{2}\left(1 + \dfrac{7x^2}{16}\right)$

$\Rightarrow \dfrac{ds}{dt} = \dfrac{1}{2}\left(1 + \dfrac{7}{12}.x^2\right).\dfrac{dx}{dt}$

put the value & $\dfrac{dA}{dt} = \dfrac{66}{7}$

Sol.18 $pr^2 = A \Rightarrow r = \sqrt{\dfrac{A}{\pi}}$

$\dfrac{dr}{dA} = \dfrac{1}{2\sqrt{A\pi}}$ & $A = 2 \times \dfrac{28}{\pi}$

$\dfrac{dr}{dA} = \dfrac{1}{2\sqrt{2 \times \dfrac{28}{11} \times \dfrac{22}{7}}} = \dfrac{1}{8}$

Now $\dfrac{dr}{dt} = \dfrac{dr}{dA} \times \dfrac{dA}{dt} = \dfrac{1}{8} \times 2 = \dfrac{1}{4}$ cm/sec

Sol.19 $\dfrac{dv}{dt} = \pi R . 2y. \dfrac{dy}{dt} - \pi y^2 . \dfrac{dy}{dt}$

Sol.20 $\dfrac{dv}{dt} \times \dfrac{1}{r} \Rightarrow \dfrac{dv}{dt} = \dfrac{c}{r} \Rightarrow \dfrac{4}{3} p . 3r^2 . \dfrac{dr}{dt} = \dfrac{c}{r}$

$\Rightarrow 4\pi \displaystyle\int r^3.dr = c\int dt \Rightarrow \pi r^4 = \underset{\pi}{\underset{\downarrow}{ct}} + \underset{\pi}{\underset{\downarrow}{k}}$

$\Rightarrow r^4 = t + 1 \qquad \Rightarrow r = (1 + t)^{1/4}$